ORIGINAL ARTICLE

Divine proportions in the growing face

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Introduction: The aim of this study was to answer this question: to what extent do facial proportions change in comparison with the divine values during growth? **Methods:** The changes of the facial proportions in ordinary subjects were analyzed with full-face photos from 20 female and 20 male subjects. In each photo, 5 transverse and 7 vertical reference distances were considered. The facial photos from childhood (age, 6.5 years), adolescence (17 years), and adulthood (30 years) of each subject were compared. For all photos in both planes (transverse and vertical), the ideal reference distances were calculated with a base value (1) and the divine proportion. These ideal distances were compared with the actual distances. Two facial disproportion indexes in the transverse and vertical planes, respectively, were determined and defined as the average absolute deviations of all measured distances from their ideal values. **Results:** The facial proportions changed, on average, only a small amount during the growth period from childhood to adulthood, but large interindividual variations were seen. The disproportion indexes in the transverse and vertical planes varied between 10.5% and 18.1% in the female subjects and 12.0% and 18.3% in the male subjects. **Conclusions:** In comparison with the divine values, facial proportions in both sexes remain rather constant during growth. (Am J Orthod Dentofacial Orthop 2008;134:472-9)

The divine proportions, or the golden ratio, was of great importance in Greek art and architecture. Phidias, a Greek sculptor, used the golden ratio so often in his work that the number 1.618 (golden ratio) was given the name *Phi*.[1] In the literature, the divine proportions were first mentioned about AD 300 by the Greek mathematician Euclid in his second and perhaps best-known book, *Elements*.[1] Some examples of divine proportions are the Parthenon, the Dionysian Procession, and Leonardo da Vinci's painting of the Mona Lisa.[2,3]

Interestingly, divine proportions can also be found in nature: the way the seed head of a sunflower is structured (ie, the arrangement of the seeds in the blossom) and the symmetry of the leaves in a pineapple or the scales of a fir cone. Even primitive forms of life, such as the spira nautilus, have a shape based on divine proportions.[4,5]

Because of the esthetically pleasing effect of divine proportions, the letter phi has become a symbol of harmony, balance, and beauty. Ricketts[4] found a relationship between divine proportions and facial beauty in young women. Heiss,[6] on the other hand, claimed that an attractive face is not necessarily based on divine proportions.

Divine proportions result from a specific geometric sectioning of a distance: a line (AB) is sectioned at point C in accordance with the golden ratio when the 2 subsections (AC to CB) correspond to each other as does the whole distance AB to the section AC. This relationship is called phi ($\varphi = 1.618$) (Fig 1).

It is obvious that the human face changes during growth.[7] There is a great difference between the craniofacial structure of an infant and an adult. In infants, the relationship between the neurocranium and the viscerocranium is 8 to 1; in adults, it has changed to 2.5 to 1. At the same time, the proportions of the cranium change because the viscerocranium grows more than the neurocranium.[8] However, it is unclear how much facial proportions change in relation to the divine values during growth.

Therefore, our aims were to answer these questions: (1) to what extent do facial proportions change from childhood to adulthood in normal, average people? (2) how do these changes relate to the divine proportions? and (3) are there differences with respect to sex?

MATERIAL AND METHODS

Our subjects were chosen from a longitudinal clinical trial at the Department of Orthodontics, Rijks University, Groningen, the Netherlands, from 1951 to 1976. Originally, 183 unselected primary schoolchildren with various occlusions (Class I to Class III) were examined. These common subjects were observed over

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Fig 1. Arithmetic expression of the golden ratio: AB/AC = AC/CB = 1.618 = φ (phi).

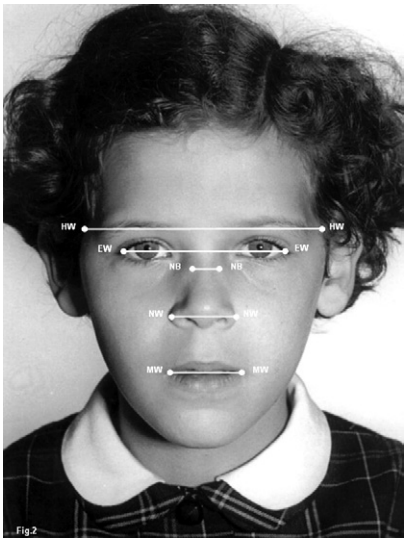

Fig 2. Reference distances in the transverse plane: *HW-HW,* head width; *EW-EW,* eye width; *NB-NB,* nose bridge width; *NW-NW,* nose width (base value); *MW-MW,* mouth width.

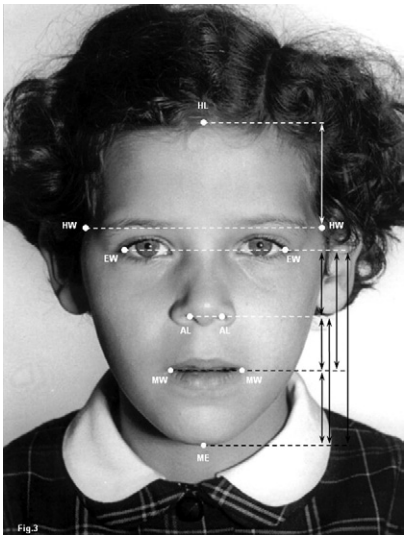

Fig 3. Reference distances in the vertical plane: *HL-HW,* forehead height; *EW-AL,* nose height; *AL-MW,* nose-lip distance (base value); *MW-ME,* chin height; *EW-MW,* upper face height; *AL-ME,* lower face height; *EW-ME,* total face height.

Table I. Phi relationship for the reference distances in the transverse plane

Transverse reference distance	*Calculation of ideal values on the basis of divine proportions*
NW-NW	Base value = 1
NB-NB	1: φ = 0.618
MW-MW	φ = 1.618
EW-EW	φ^2 = 2.618
HW-HW	φ^3 = 4.236

Table II. Phi relationship for the reference distances in the vertical plane

Vertical reference distance	*Calculation of ideal values on the basis of divine proportions*
AL-MW	Base value = 1
EW-AL	φ = 1.618
MW-ME	φ = 1.618
HL-HW	φ^2 = 2.618
AL-ME	φ^2 = 2.618
EW-MW	φ^2 = 2.618
EW-ME	φ^3 = 4.236

Table III. Method error of all transverse and vertical reference distances at F1 (average age, 6.5 years) and F3 (average age, 30 years)

	Male subjects		*Female subjects*	
Reference distance (mm)	*F1 (n = 20)*	*F3 (n = 20)*	*F1 (n = 20)*	*F3 (n = 20)*
HW-HW	0.10	0.04	0.12	0.14
EW-EW	0.13	0.16	0.15	0.06
NB-NB	0.24	0.24	0.22	0.13
MW-MW	0.12	0.13	0.08	0.11
MW-ME	0.10	0.09	0.05	0.07
AL-ME	0.08	0.10	0.05	0.07
HL-HW	0.22	0.20	0.21	0.20
EW-AL	0.09	0.10	0.08	0.06
EW-MW	0.07	0.05	0.04	0.05
EW-ME	0.07	0.10	0.05	0.08

25 years. At the last recall in 1976, there were still 100 subjects participating in the study.

At the first examination, 1951 to 1954, the subjects were between 5.75 and 7.5 years of age. At the second examination, 1963 to 1965, they were between 15.5 and 18.75 years. At the third examination, 1975 to 1976, they were between 28.25 and 31.75 years.

From this comprehensive material, full-face photos of the subjects with closed or almost closed lips were chosen. It was important that all reference points could be clearly seen on the photos. In total, 102 full-face photos of 40 subjects (20 female, 20 male) were

Table IV. Average percentage deviations of all distances from the ideal values calculated on the basis of divine proportions and the disproportion indexes for all distances

Variable	*Average deviation (%) of divine proportions at F1*		*Average deviation (%) of divine proportions at F2*		*Average deviation (%) of divine proportions at F3*	
Reference distance	*Females (n = 20)*	*Males (n = 20)*	*Females (n = 10)*	*Males (n = 12)*	*Females (n = 20)*	*Males (n = 20)*
Transverse						
HW-HW	−9.2	−11.7	−12.7	−14.5	−14.3	−15.0
EW-EW	−4.7	−11.5	−1.5	−11.1	−4.9	−11.7
NB-NB	−29.6	−28.7	−35.3	−30.9	−35.0	−31.3
MW-MW	−21.2	−22.3	−15.6	−17.7	−15.1	−16.7
Transverse disproportion indexes	18.1	16.9	16.6	18.3	17.5	18.1
Vertical						
MW-ME	−10.8	−12.9	−4.5	+2.5	−6.3	−5.8
AL-ME	−7.7	−9.4	−8.5	+2.1	−8.3	−3.7
HL-HW	−21.8	−25.5	−25.2	−24.8	−23.3	−22.0
EW-AL	−24.8	−25.5	−26.5	−24.0	−22.2	−20.8
EW-MW	−17.6	−16.1	−17.0	−16.6	−15.3	−13.2
EW-ME	−16.6	−16.6	−12.9	−9.5	−14.2	−11.3
Vertical disproportion indexes	10.5	16.7	16.3	12.0	15.8	13.7

The values of groups A (10 female, 12 male subjects) and B (20 female, 20 male subjects) at the average ages of 6.5 years (F1), 17 years (F2), and 30 years (F3) are shown. Negative values indicate reduced values compared with the ideal value (0).

suitable for the analysis of the facial proportions. Photos at the following stages were available: F1, average age 6.5 years; F2, average age 17 years; and F3, average age 30 years.

With respect to the availability of photos (F1, F2, and F3), the subjects were divided into groups A and B. Group A included 10 female subjects and 12 males subjects with photos at all stages. Group B included 20 female subjects and 20 males subjects with photos at F1 and F3.

Five transverse and 7 vertical reference distances were defined for the calculation of the facial proportions. The reference points corresponded for the most part to those of Ricketts.[4] They are shown in Figure 2 (transverse plane) and Figure 3 (vertical plane), with their corresponding reference distances. As base values (1), the reference distance NW-NW in the transverse plane and the reference distance AL-MW in the vertical plane were used. If a reference point could not be identified clearly on a photo—eg, because of the subject's hairstyle—the visible contralateral point was taken for reference. All photos were evaluated twice. The resulting mean values were used for the final calculations.

With the 2 base values for the transverse and vertical planes, the ideal values for all reference distances were calculated by using the phi relationship ($\varphi = 1.618$). These measured distances were then compared with the ideal values, and the percentage differences were calculated.

Tables I and II show the phi relationships for the transverse and vertical reference distances, respectively.

In the transverse and the vertical planes, a disproportion index was created for each photo (Heiss[6]). The index is the quotient of the sum of the absolute percentage deviations of all measured distances from the divine values and the number of reference distances. There were 4 reference distances in the transverse plane and 6 reference distances in the vertical plane. Calculations were made for all 102 full-face photos.

In the evaluation of the disproportion indexes, a clinically nonrelevant area of ± 5% in relation to the ideal values was determined. Subjects with disproportion values in this range were not considered to be disproportional.

By using the duplicate registrations, the method error (locating and marking the reference points and measuring the reference distances) was calculated for each reference distance.

The following formula was used for the method error (ME) calculations[9]:

$$ME = \sqrt{\frac{\sum d^2}{2n}}$$

where d is the difference between the 2 measured values and n is the number of subjects. Table III shows the method error calculations for all subjects.

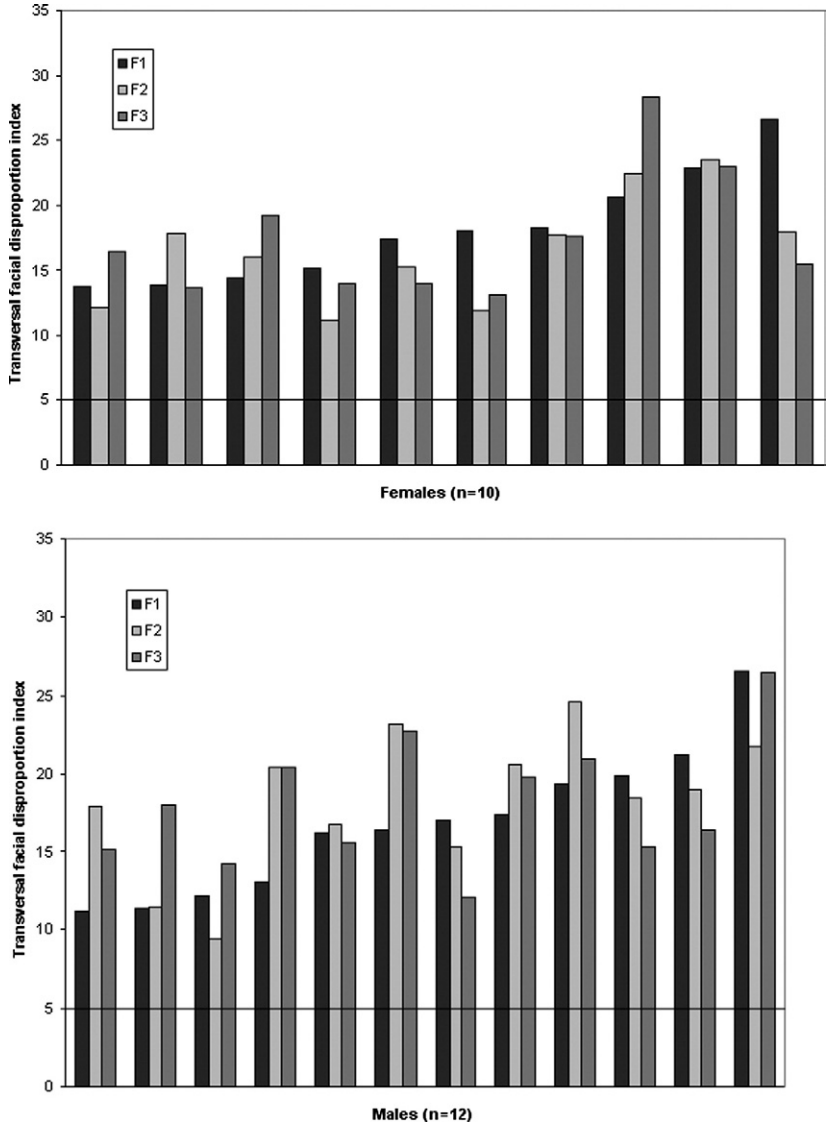

Fig 4. Transverse facial disproportion index in group A subjects: average absolute deviations of all transverse reference distances from their ideal value (0) in 10 female subjects (*top*) and 12 male subjects (*bottom*). *F1,* first photo; *F2,* second photo; *F3,* third photo. The *line* inside the diagram defines the clinically nonrelevant area of 5%.

RESULTS

The disproportion index values for all transverse reference distances in the female subjects were 18.1% at F1, 16.6% at F2, and 17.5% at F3. In the male subjects, the index values were 16.9% at F1, 18.3% at F2, and 18.1% at F3 (Table IV).

The disproportion index values for all vertical reference distances in the female subjects were 10.5% at F1, 16.3% at F2, and 15.8% at F3. In the male subjects, the index values were 16.7% at F1, 12.0% at F2 and 13.7% at F3 (Table IV).

There was much intraindividual and interindividual variation of the facial disproportion indexes in the transverse (Fig 4) and vertical (Fig 5) planes. To demonstrate this variation, the facial photos and their corresponding disproportion indexes of 4 subjects are shown in Figures 6 through 10.

Subject 1 (Figs 6 and 10) was a boy, aged 6.5 years at F1, 15.75 years at F2, and 28.25 years at F3. In the transverse plane, the disproportion index increased steadily with age (F1, 15.2%; F2, 20.5%; F3, 22.0%). In the vertical plane, the corresponding indexes (F1, 6.8%;

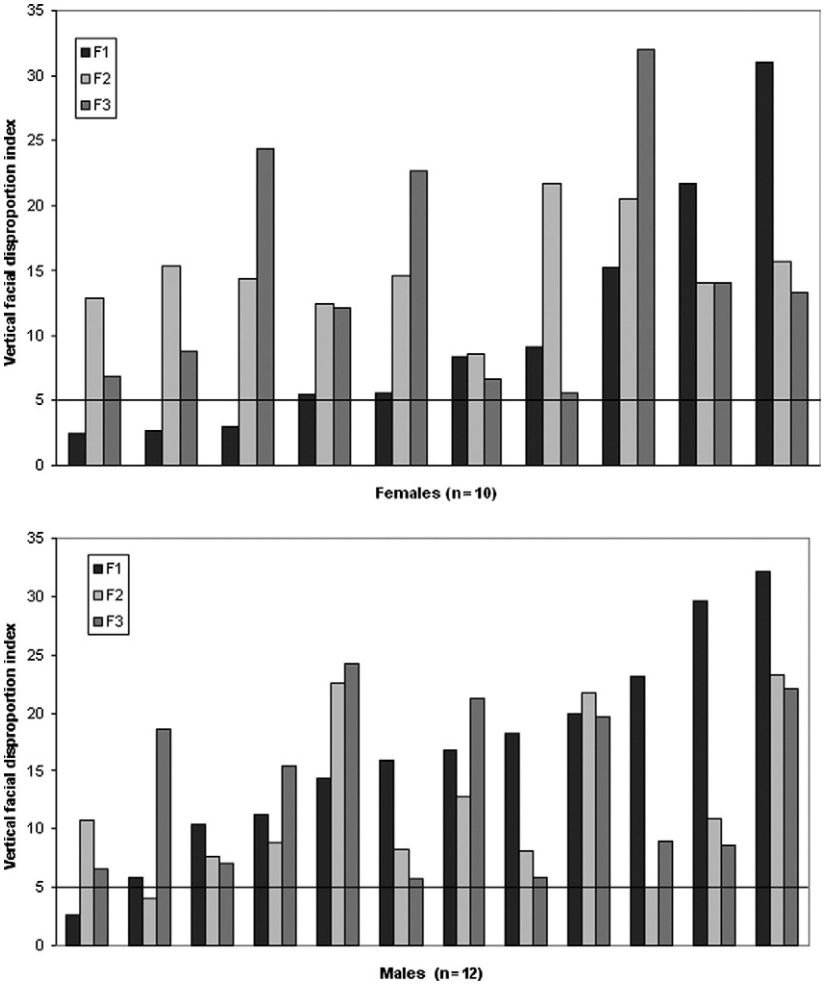

Fig 5. Vertical facial disproportion index in group A subjects: average deviation of all vertical reference distances from their ideal value (0) in 10 female subjects (*top*) and 12 male subjects (*bottom*). *F1,* first photo; *F2,* second photo; *F3,* third photo. The *line* inside the diagram defines the clinically nonrelevant area of 5%.

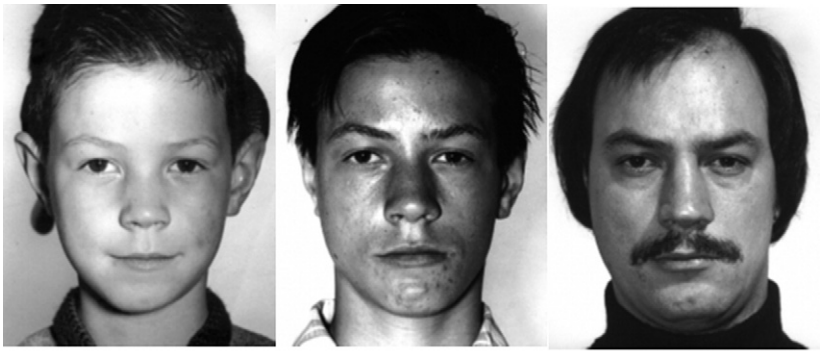

Fig 6. Facial photos of subject 1 at ages 6.5 years (*F1*), 15.75 years (*F2*), and 28.25 years (*F3*).

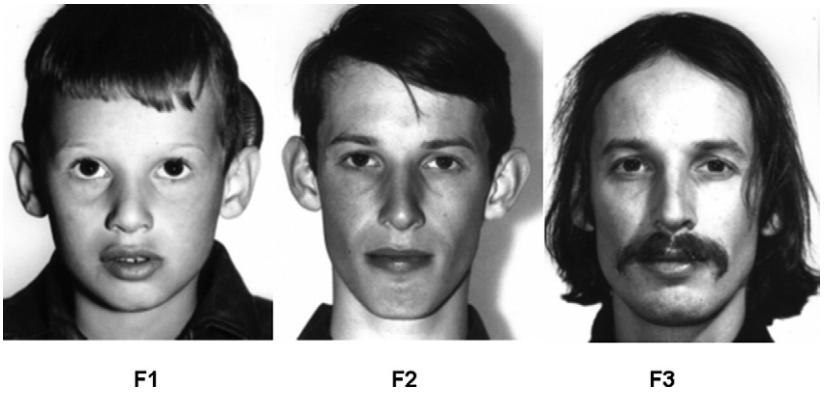

Fig 7. Facial photos of subject 2 at ages 7 years (*F1*), 17.75 years (*F2*), and 28.5 years (*F3*).

Fig 8. Facial photos of subject 3 at ages 7 years (*F1*), 18 years (*F2*), and 28.75 years (*F3*).

Fig 9. Facial photos of subject 4 at ages 7.25 years (*F1*), 16.25 years (*F2*), and 28.75 years (*F3*).

F2, 11.0%; F3, 7.5%) were lower than in the transversal plane, with only slight deviations from the ideal values.

Subject 2 (Figs 7 and 10) was a boy, aged 7 years at F1, 17.75 years at F2, and 28.5 years at F3. In the transversal plane, all 3 photos showed similar deviations of the disproportion indexes in relation to the ideal values at all examinations (F1, 14.0%; F2, 12.5%; F3, 13.6%). In the vertical plane, the dispro-

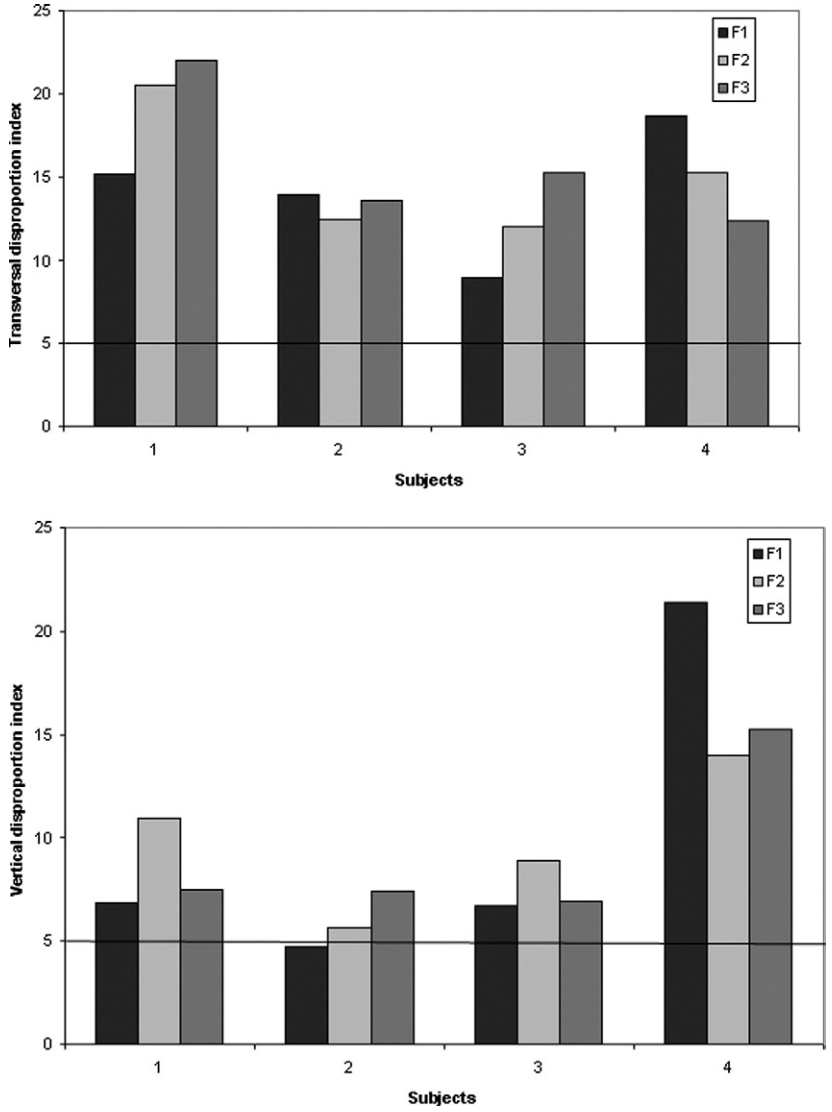

Fig 10. Transverse and vertical disproportion indexes of subjects 1-4 (Figs 6-9). The 0 line defines the ideal value. *F1,* first photo; *F2,* second photo; *F3,* third photo. The *lines* inside the diagrams define the clinically nonrelevant area of 5%.

portion indexes were lower than in the transverse plane but increased continuously with age (F1, 4.7%; F2, 5.6%; F3, 7.4%).

Subject 3 (Figs 8 and 10) was a girl, aged 7 years at F1, 18 years at F2, and 28.75 years at F3. In the transverse plane, the disproportion index steadily increased with age (F1, 8.9%; F2, 12.1%; F3, 15.3%). In the vertical plane, the index was similar at all examinations (F1, 6.7%; F2, 8.9%; F3, 6.9%).

Subject 4 (Figs 9 and 10) was a girl, aged 7.25 years at F1, 16.25 years at F2, and 28.75 years at F3. In the transverse plane, the disproportion index continuously decreased with age (F1, 18.7%; F2, 15.3%; F3, 12.4%). In the vertical plane, the disproportion index was much larger at F1 (21.4%) than at F2 (14.0%) and F3 (15.3%).

DISCUSSION

The idea for this study came from Ricketts[4] and Heiss.[6] However, their investigations focused on beautiful faces, whereas ours dealt with average faces. We aimed to evaluate how much facial proportions change during growth. The subjects of the Groningen study represent a cross-section of the total population, and none had orthodontic care during the documentation

period. Thus, treatment-related facial-proportion changes could be ruled out.

To reduce the method error, all photos were measured twice, and the mean value was used for the final evaluations. This was important, because some reference points and distances were difficult to define and measure. Most difficult were nose bridge width in the transverse plane and forehead height in the vertical plane.

Because children have facial features that are no longer found in adolescents and adults, changes in facial proportions are expected from growth.[7] However, this study showed that the facial proportions changed only minimally. In all 3 age groups (F1, F2, F3), the deviations of the disproportion indexes from the divine values were similar in the transverse and vertical planes. Since there were no perceptible sex-related differences, it could be hypothesized that the facial proportions of both men and women develop similarly during the different age phases. Thus, facial proportions seem to be predetermined already in childhood, with only slight variations during growth. But there were great interindividual variations. That is clearly shown by the 4 subjects (Figs 6-10).

This study indicates also that divine facial proportions are rarely achieved in ordinary people. The average percentage deviations from the ideal values of all transverse and vertical reference distances (disproportion index) varied between 10.5% and 18.1% for the female subjects and 12.0% and 18.3% for the male subjects. In the study of Heiss,[6] who analyzed the beautiful faces of cover models, the respective deviations were only between 2.6% and 4.4% for the women and 4.4% and 5.0% for the men. Thus, it appears that people with attractive faces show greater concordance with the divine proportions than do those with common faces. In contrast to Heiss,[6] we did not consider subjects with a clinically nonrelevant disproportion index deviation of ± 5% from the ideal value. This implies that subjects with almost divine facial proportions were not included in the evaluation.

CONCLUSIONS

1. The transverse and vertical facial proportions changed, on average, only slightly from childhood to adulthood. There was, however, great variation between subjects.
2. The disproportion indexes, which represent the average percentage deviation of all measured reference distances from the corresponding divine (ideal) values, in the transverse and vertical planes varied between 10.5% and 18.1% in female subjects and between 12.0% and 18.3% in male subjects.
3. The facial disproportion indexes seemed to be independent of sex.

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